

# Explaining $\Omega_{\text{baryon}} \approx 0.2\Omega_{\text{dark}}$ through the synthesis of ordinary matter from mirror matter: A more general analysis

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The emerging cosmological picture is of a spatially flat universe composed predominantly of three components: ordinary baryons ( $\Omega_B \approx 0.05$ ), nonbaryonic dark matter ( $\Omega_{\text{dark}} \approx 0.22$ ) and dark energy ( $\Omega_\Lambda \approx 0.7$ ). We recently proposed that ordinary matter was synthesized from mirror matter, motivated by the argument that the observed similarity of  $\Omega_B$  and  $\Omega_{\text{dark}}$  suggests an underlying similarity between the fundamental properties of ordinary and dark matter particles. In this paper we generalize the previous analysis by considering a wider class of effective operators that nongravitationally couple the ordinary and mirror sectors. We find that while all considered operators imply  $\Omega_{\text{dark}} = \text{few} \times \Omega_B$ , only a subset quantitatively reproduce the observed ratio  $\Omega_B/\Omega_{\text{dark}} \approx 0.20$ . The  $\sim 1$  eV mass scale induced through these operators hints at a connection with neutrino oscillation physics.

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## I. INTRODUCTION

A variety of evidence, culminating in the Wilkinson Microwave Anisotropy Probe (WMAP) measurements of the cosmic microwave background [1], points to a spatially flat universe composed of approximately 5% baryons ( $B$ ), 22% nonbaryonic dark matter (DM) and roughly 70% dark energy. These fractions, while different, are suspiciously similar in magnitude. From a theoretical point of view, this is surprising, as one might *a priori* expect the physics of each component to be rather different. This is a new naturalness puzzle.

The similarity of the positive and negative pressure components, roughly 30% and 70% respectively, has received some attention in the literature. However, the similar magnitudes of  $\Omega_B$  and  $\Omega_{\text{dark}}$  are also puzzling. (As usual,  $\Omega_X$  denotes the ratio of the energy density of component  $X$  and the critical density.) In Ref. [2], we began an exploration of a possible solution. We proposed that the similar baryonic and dark matter densities suggest that the internal microphysics of each component are actually similar or identical. Specifically, this will be case if the dark matter is identified with mirror baryonic matter,  $B'$ .<sup>1</sup> The purpose of this new paper is to extend the analysis of Ref. [2].

A theoretical motivation for mirror matter [4,5] is to retain the full Poincaré group, including improper Lorentz transformations such as parity inversion and time reversal, as an exact symmetry group of nature, despite the  $V-A$  character of weak interactions. Mirror matter theories have a  $G \otimes G$  gauge group structure, with ordinary and mirror particles assigned to  $(R,1)$  and  $(1,R)$  representations, respectively. Un-

der parity transformations, a given ordinary particle interchanges with its mirror partner. For fermions, a left-handed (right-handed) ordinary particle is paired with a right-handed (left-handed) mirror partner. Time-reversal invariance  $T'$  follows from the  $CPT$  theorem, with  $T'P' \equiv CPT$  where  $P'$  is the exact mirror parity operation defined above. We shall focus on the simplest mirror matter model, where  $G$  is the standard model gauge group, there are no exotic fermions in addition to the mirror partners of known quarks and leptons, and there is just one Higgs doublet paired with one mirror Higgs doublet. We shall consider only the case where the vacuum also respects improper Lorentz transformations, that is, the discrete spacetime symmetries are not spontaneously broken [5]. This means that an ordinary particle and its mirror partner have the same mass. A large region of Higgs potential parameter space allows this aesthetically appealing scenario [5].

Mirror particles interact amongst themselves through their own versions of the strong, weak and electromagnetic interactions. The two sectors are inevitably coupled by gravity, but various nongravitational interactions can also connect them. Within the class of renormalizable, gauge-invariant and mirror-symmetry-invariant operators, these interactions are photon-mirror-photon kinetic mixing and Higgs-boson-mirror-Higgs-boson coupling through a  $\phi^\dagger \phi \phi'^\dagger \phi'$  term (we denote mirror fields with a prime) [5]. The implications of these interactions have been explored in several papers [5,6].

The mechanism under study here employs dimension-5 effective operators of the form *lepton-Higgs-boson-lepton'*–*Higgs-boson'* to couple the ordinary and mirror sectors. It also requires an assumption about the initial lepton and baryon asymmetries (prior to reprocessing). In Ref. [2], a particular such operator with natural initial asymmetries (see below for a review) was found to yield

$$\frac{\Omega_B}{\Omega_{\text{dark}}} \approx 0.20-0.21, \quad (1)$$

in excellent agreement with the allowed range suggested by WMAP [1]  $0.20 \pm 0.02$ .

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<sup>1</sup>The  $\Omega_B \sim \Omega_{\text{dark}}$  issue was very briefly touched upon by Bento and Berezhiani [3] also in the context of mirror matter models. Note, though, that their mechanism produces  $\Omega_B = \Omega'_B$  for the symmetric mirror model.

We would now like to know if this encouraging quantitative success is unique to the effective operator we happened to consider in Ref. [2]. The purpose of this paper is to examine a wider class of operators. We shall find that while all selected operators yield  $\Omega_B/\Omega_{\text{dark}} < 1$ , only a subset are quantitatively consistent with the observations, and that the operator of Ref. [2] is not uniquely favored (from this cosmological perspective).

The next section reviews the dynamical framework of Ref. [2] and the section after that presents our calculations. We end by discussing our results and drawing conclusions.

## II. REVIEW OF THE DYNAMICAL FRAMEWORK

A fundamental feature of mirror matter cosmology is that the temperature of the mirror sector,  $T'$ , is expected to be different from that of the ordinary sector,  $T$ , at least during certain epochs. For the late epoch during which big bang nucleosynthesis takes place,  $T'/T \lesssim 0.5$  should hold in order to constrain the expansion rate of the universe at that time. From a later period still, large scale structure formation with mirror DM suggests a slightly more stringent constraint,  $T'/T \lesssim 0.2$  [7]. The smaller this ratio is, the more does mirror DM resemble standard cold DM during the linear regime of density perturbation growth (they obviously must differ in the nonlinear regime because mirror DM is self-interacting, chemically complex and dissipative) [7]. For earlier epochs, however, we have no observational constraints that  $T'/T$  must satisfy. If  $\Omega_{\text{dark}} = \Omega'_B$ , our fundamental hypothesis, then the inequality of  $\Omega'_B$  and  $\Omega_B$  strongly suggests that the temperatures were also different for at least part of the time during baryogenesis.

A temperature difference can be created, for example, through inflation [8]. Imagine that there is an inflaton and a mirror inflaton, and that inflation is seeded by a fluctuation through whichever of these fields the fluctuation “favors.” Upon reheating, the *macroscopic* ordinary-mirror asymmetry generated by the amplified fluctuation will translate into  $T' \neq T$  provided the two sectors are weakly enough coupled to each other. The subsequent evolution of  $T'/T$  depends on the precise nature of the ordinary-mirror coupling terms. We emphasize that the macroscopic temperature asymmetry is not at all inconsistent with an exactly symmetric microphysics.

We can now review the main dynamic events our scenario requires:

*Step 1.* Suppose that reheating after inflation leaves a universe with

$$T' \gg T, \quad (2)$$

that is, the universe is totally dominated initially by mirror matter. This would happen if inflation was driven by the vacuum energy of a mirror inflaton.

*Step 2.* Around a certain temperature,  $T' = T_1$ , mirror baryon and/or mirror lepton asymmetries are created. We prefer not to specify the mechanism. It might be the out-of-equilibrium decays of the heavy neutral mirror leptons as per

leptogenesis [9], or something else. No significant ordinary-sector asymmetries are generated—a reasonable assumption in the limit of Eq. (2).

*Step 3.* These initial asymmetries are chemically reprocessed. In particular, ordinary asymmetries are created through effective dimension-5 operators of the form

$$L = \left( \frac{1}{M_N} \right)_{ij} \bar{\ell}_{iL} \phi^c \ell'_{jR} \phi' + \text{H.c.}, \quad (3)$$

where  $\ell_L$  is a left-handed ordinary lepton doublet,  $\ell'_R$  is its mirror partner,  $\phi$  is the ordinary Higgs doublet, and  $\phi'$  is its mirror partner. The indices  $i, j = 1, 2, 3$  denote the three families. These are the lowest dimension nonrenormalizable, gauge-invariant operators one can construct out of the fundamental fields. They may be generated from renormalizable operators through the exchange of gauge-singlet fermions, hence the notation  $M_N$ . They also induce terms  $m_\nu \equiv \langle \phi \rangle^2 / M_N$  in the light neutrino mass matrix. As well as contributing (with sphaleron and other effects) to the chemical reprocessing of asymmetries, these interactions also thermally equilibrate the sectors, that is, induce  $T = T'$ . They operate during the temperature regime

$$M_N \gtrsim T \gtrsim T_2 \equiv 10^{10} (eV/m_\nu)^2 \text{ GeV}. \quad (4)$$

For  $T \gtrsim M_N$ , the effective Lagrangian approach is not valid and the parent fundamental, renormalizable interactions are slower than the expansion rate, while for  $T \lesssim T_2$  the effective interactions are themselves slower than the expansion rate. The issue explored in this paper is how different family hierarchy assumptions for the  $(1/M_N)_{ij}$  affect  $\Omega_B/\Omega'_B$ .

*Step 4.* A second but relatively brief inflationary episode (or some alternative process) must then occur (beginning at some temperature,  $T_3$ ) in order to set up the mild hierarchy,  $T'/T \lesssim 0.2-0.5$ , as required for big bang nucleosynthesis and the later linear perturbation growth periods. It is interesting that this hierarchy is in the opposite sense to that created by the first inflationary episode [see Eq. (2)]. If the mirror inflaton induces the earlier inflationary burst, then it is tempting to ascribe the later burst to a largely failed attempt by the ordinary inflaton to reciprocate.

## III. CALCULATING $\Omega_B/\Omega_{\text{dark}}$

The main issue is the family structure assumed for Eq. (3). We shall take for simplicity and definiteness that one flavor combination for the effective operators dominates [through having the largest  $(1/M_N)_{ij}$ ], with all the others too small to affect the chemical reprocessing. Given the connection with neutrino mass, we shall assume the relevant  $M_N$  is of order  $\langle \phi \rangle^2 / (1 \text{ eV})$ , since the  $\sim 1 \text{ eV}$  scale is a reasonable upper “bound” on neutrino mass, and is even directly suggested by the Liquid Scintillating Neutrino Detector (LSND) anomaly [10] [see also Ref. [11] for discussions of ordinary-mirror neutrino mixing].

In Ref. [2], we examined only one case, where the  $i = j = 2$  term dominates. Assuming that there is just one eV mass term connecting the ordinary and mirror sectors and that it is approximately flavor diagonal, then there are just 6 distinct

cases to consider (including  $i=j=2$  to be called case 1 from now on):

$$\text{case 1: } L = \frac{1}{M_N} \bar{\ell}_{2L} \phi^c \ell'_{2R} \phi' + \text{H.c.}, \quad (5)$$

$$\text{case 2: } L = \frac{1}{M_N} \bar{\ell}_{1L} \phi^c \ell'_{1R} \phi' + \text{H.c.}, \quad (6)$$

$$\text{case 3: } L = \frac{1}{M_N} \bar{\ell}_{3L} \phi^c \ell'_{3R} \phi' + \text{H.c.}, \quad (7)$$

$$\begin{aligned} \text{case 4: } L = & \frac{1}{M_N} \bar{\ell}_{2L} \phi^c \ell'_{3R} \phi' \\ & + \frac{1}{M_N} \bar{\ell}'_{2R} \phi'^c \ell_{3L} \phi + \text{H.c.}, \end{aligned} \quad (8)$$

$$\begin{aligned} \text{case 5: } L = & \frac{1}{M_N} \bar{\ell}_{1L} \phi^c \ell'_{3R} \phi' \\ & + \frac{1}{M_N} \bar{\ell}'_{1R} \phi'^c \ell_{3L} \phi + \text{H.c.}, \end{aligned} \quad (9)$$

$$\begin{aligned} \text{case 6: } L = & \frac{1}{M_N} \bar{\ell}_{1L} \phi^c \ell'_{2R} \phi' \\ & + \frac{1}{M_N} \bar{\ell}'_{1R} \phi'^c \ell_{2L} \phi + \text{H.c.} \end{aligned} \quad (10)$$

These operators affect chemical reprocessing by constraining the chemical potentials of the species concerned. We now need to review how the reprocessing is analyzed.

For temperatures below about  $10^{12}$  GeV, QCD [12] and electroweak [13] nonperturbative processes plus the Yukawa interactions for the fermions  $c$ ,  $\tau$ ,  $b$  and  $t$  are faster than the expansion rate of the universe [14]. The Yukawa interactions for  $e_R$ ,  $u_R$ ,  $d_R$ ,  $\mu_R$  and  $s_R$  do not become fast enough until the temperature drops below about  $10^{10}$  GeV. Diagonal and off-diagonal weak interactions involving left-handed quarks are also happening rapidly. By  $T_2 \approx 10^{10}$  GeV [see Eq. (4)], the selected effective operator, one of Eqs. (5)–(10), has also induced rapid interactions, affecting the chemical composition and inducing  $T=T'$ . At about  $T=10^{10}$  GeV, all the quarks, leptons, Higgs bosons and their mirror partners are in thermal equilibrium with distribution functions governed by the temperature and chemical potentials for all the involved species. We denote the chemical potential for species  $X$  by  $\mu_X$  ( $X'$  by  $\mu'_X$ ). The rapid processes listed above relate the  $\mu$ 's. In addition, we impose electric charge or hypercharge neutrality, and mirror electric/hypercharge neutrality, for the universe.

The chemical constraint equations are [14]

$$9\mu_q + \sum_{i=1}^3 \mu_{\ell_i} = 0 \quad (\text{electroweak nonperturbative}),$$

$$6\mu_q - \sum_{i=1}^3 (\mu_{u_i} + \mu_{d_i}) = 0 \quad (\text{QCD nonperturbative}),$$

$$3\mu_q + 2\mu_\phi + \sum_{i=1}^3 (2\mu_{u_i} - \mu_{d_i} - \mu_{\ell_i} - \mu_{e_i}) = 0$$

(electric charge neutrality),

$$\mu_q - \mu_\phi - \mu_{d_3} = 0 \quad (b\text{-quark Yukawa}),$$

$$\mu_q + \mu_\phi - \mu_{u_2} = 0 \quad (c\text{-quark Yukawa}),$$

$$\mu_q + \mu_\phi - \mu_{u_3} = 0 \quad (t\text{-quark Yukawa}),$$

$$\mu_{\ell_3} - \mu_\phi - \mu_{e_3} = 0 \quad (\tau\text{-lepton Yukawa}), \quad (11)$$

plus the corresponding seven equations from the mirror sector. The simplified notation here is  $\mu_q \equiv \mu_{q_{1L}} = \mu_{q_{2L}} = \mu_{q_{3L}}$ , where the equalities are enforced by off-diagonal weak interactions;  $\mu_{u_i} \equiv \mu_{u_{iR}}$ ,  $\mu_{d_i} \equiv \mu_{d_{iR}}$  and  $\mu_{\ell_i} \equiv \mu_{\ell_{iL}}$ . We are working in the Yukawa-diagonal basis (so  $u_{3R}$  becomes the right-handed component of the mass eigenstate  $t$  quark after the electroweak phase transition, and so on).

Then there are one or two more constraint equations, induced by the dimension-5 operator(s):

$$\text{case 1: } -\mu_{\ell_2} - \mu_\phi + \mu'_{\ell_2} + \mu_{\phi'} = 0, \quad (12)$$

$$\text{case 2: } -\mu_{\ell_1} - \mu_\phi + \mu'_{\ell_1} + \mu_{\phi'} = 0, \quad (13)$$

$$\text{case 3: } -\mu_{\ell_3} - \mu_\phi + \mu'_{\ell_3} + \mu_{\phi'} = 0, \quad (14)$$

$$\text{case 4: } -\mu_{\ell_2} - \mu_\phi + \mu'_{\ell_3} + \mu_{\phi'} = 0;$$

$$-\mu'_{\ell_2} - \mu'_\phi + \mu_{\ell_3} + \mu_\phi = 0, \quad (15)$$

$$\text{case 5: } -\mu_{\ell_1} - \mu_\phi + \mu'_{\ell_3} + \mu_{\phi'} = 0;$$

$$-\mu'_{\ell_1} - \mu'_\phi + \mu_{\ell_3} + \mu_\phi = 0, \quad (16)$$

$$\text{case 6: } -\mu_{\ell_1} - \mu_\phi + \mu'_{\ell_2} + \mu_{\phi'} = 0;$$

$$-\mu'_{\ell_1} - \mu'_\phi + \mu_{\ell_2} + \mu_\phi = 0. \quad (17)$$

So cases 1–3 have 15 constraint equations, while cases 4–6 have 16. With 28 chemical potential variables, this leaves 13 free  $\mu$ 's for cases 1–3, and 12 for cases 4–6. The free variables correspond to conserved quantities.

The solution strategy is simply to (i) select the appropriate number of  $\mu$ 's (13 or 12) as independent variables, (ii) solve for the remaining chemical potentials in terms of them, (iii) identify the independent, conserved quantities for each case

TABLE I. The  $\alpha$  and  $\beta$  coefficients in the expansions of  $B$  and  $L$  for cases 1 and 2 as defined in Eqs. (19) and (20).

$\alpha_0 = \frac{69}{316}$		$\beta_0 = \frac{-89}{316}$	
$\alpha_1 = \frac{43263}{98276}$	$\alpha'_1 = \frac{-345}{98276}$	$\beta_1 = \frac{-55803}{98276}$	$\beta'_1 = \frac{445}{98276}$
$\alpha_2 = \frac{7050}{24569}$	$\alpha'_2 = \frac{414}{24569}$	$\beta_2 = \frac{-16571}{24569}$	$\beta'_2 = \frac{-534}{24569}$
$\alpha_3 = \frac{45189}{98276}$	$\alpha'_3 = \frac{-6003}{98276}$	$\beta_3 = \frac{31443}{98276}$	$\beta'_3 = \frac{7743}{98276}$
$\alpha_4 = \frac{23385}{98276}$	$\alpha'_4 = \frac{15801}{98276}$	$\beta_4 = \frac{59567}{98276}$	$\beta'_4 = \frac{-20381}{98276}$
$\alpha_5 = \frac{-963}{24569}$	$\alpha'_5 = \frac{2829}{24569}$	$\beta_5 = \frac{5515}{24569}$	$\beta'_5 = \frac{-3649}{24569}$
$\alpha_6 = \frac{-963}{49138}$	$\alpha'_6 = \frac{2829}{49138}$	$\beta_6 = \frac{5515}{49138}$	$\beta'_6 = \frac{-3649}{49138}$

and write the associated chemical potential as a linear combination of the independent chemical potentials chosen at step (i). Finally, (iv) solve for the baryon/lepton and mirror baryon/lepton asymmetries at  $T \approx 10^{10}$  GeV in terms of the conserved charges. Below  $10^{10}$  GeV, other processes come into play in determining the final, low temperature values of the baryon and mirror baryon asymmetries via a by now standard procedure (see below).

We now examine each case defined by Eqs. (5)–(10) separately. In the main text, we shall write down the conserved charges and give the expressions for  $B$ ,  $L$ ,  $B'$  and  $L'$  at  $T \approx 10^{10}$  GeV in terms of those charges. In the Appendix, we give the algebraic details for case 1 to illustrate the methodology.

#### A. Cases 1 and 2

Considering first case 1, at  $T \sim 10^{10}$  GeV there are 13 conserved charges:

$$\begin{aligned}
 \mathcal{L}_0 &= \frac{1}{3}B - L_2 + \frac{1}{3}B' - L'_2, \\
 \mathcal{L}_1 &= \frac{1}{3}B - L_1, \quad \mathcal{L}'_1 = \frac{1}{3}B' - L'_1, \\
 \mathcal{L}_2 &= \frac{1}{3}B - L_3, \quad \mathcal{L}'_2 = \frac{1}{3}B' - L'_3, \\
 \mathcal{L}_3 &= L_{e_{1R}}, \quad \mathcal{L}'_3 = L'_{e_{1R}}, \\
 \mathcal{L}_4 &= L_{e_{2R}}, \quad \mathcal{L}'_4 = L'_{e_{2R}}, \\
 \mathcal{L}_5 &= B_{u_{1R}} - B_{d_{1R}}, \quad \mathcal{L}'_5 = B'_{u_{1R}} - B'_{d_{1R}},
 \end{aligned}$$

$$\mathcal{L}_6 = B_{d_{1R}} - B_{d_{2R}}, \quad \mathcal{L}'_6 = B'_{d_{1R}} - B'_{d_{2R}}. \quad (18)$$

The notation is that  $L_i$  is the lepton number of family  $i$ ,  $B$  ( $L = L_1 + L_2 + L_3$ ) is the total baryon (lepton) number,  $B_{q_{iR}}$  is the charge for right-handed quark  $q$  from family  $i$ , and  $L_{e_{iR}}$  is the charge for the right-handed charged lepton of family  $i$ . The corresponding quantities from the mirror sector carry a prime.

In terms of these quantities,

$$\begin{aligned}
 B &= \alpha_0 \mathcal{L}_0 + \sum_{i=1}^6 \alpha_i \mathcal{L}_i + \sum_{i=1}^6 \alpha'_i \mathcal{L}'_i, \\
 L &= \beta_0 \mathcal{L}_0 + \sum_{i=1}^6 \beta_i \mathcal{L}_i + \sum_{i=1}^6 \beta'_i \mathcal{L}'_i. \quad (19)
 \end{aligned}$$

Under mirror symmetry,  $B \leftrightarrow B'$ ,  $L \leftrightarrow L'$ ,  $\mathcal{L}_i \leftrightarrow \mathcal{L}'_i$  (and  $\mathcal{L}_0 \rightarrow \mathcal{L}_0$ ). Hence,

$$\begin{aligned}
 B' &= \alpha_0 \mathcal{L}_0 + \sum_{i=1}^6 \alpha_i \mathcal{L}'_i + \sum_{i=1}^6 \alpha'_i \mathcal{L}_i, \\
 L' &= \beta_0 \mathcal{L}_0 + \sum_{i=1}^6 \beta_i \mathcal{L}'_i + \sum_{i=1}^6 \beta'_i \mathcal{L}_i. \quad (20)
 \end{aligned}$$

The values of the  $\alpha$  and  $\beta$  parameters are given in Table I.

Results for case 2 follow from case 1 with the replacements  $L_2 \rightarrow L_1$ ,  $L'_2 \rightarrow L'_1$  in  $\mathcal{L}_0$  and  $L_1$  ( $L'_1$ )  $\rightarrow L_2$  ( $L'_2$ ) in  $\mathcal{L}_1$  ( $\mathcal{L}'_1$ ). Because both muon- and electron-Higgs-boson Yukawa interactions are negligible above  $10^{10}$  GeV, the  $\alpha, \beta$  coefficients for case 2 are trivially the same as those for case 1 which were given in Table I.

TABLE II. The  $\alpha$  and  $\beta$  coefficients in the expansions of  $B$  and  $L$  for case 3.

$\alpha_0 = \frac{12}{79}$		$\beta_0 = \frac{-55}{158}$	
$\alpha_1 = \alpha_2 = \frac{735}{1738}$	$\alpha'_1 = \alpha'_2 = \frac{12}{869}$	$\beta_1 = \beta_2 = \frac{-42}{79}$	$\beta'_1 = \beta'_2 = \frac{-5}{158}$
$\alpha_3 = \alpha_4 = \frac{71}{158}$	$\alpha'_3 = \alpha'_4 = \frac{-4}{79}$	$\beta_3 = \beta_4 = \frac{67}{237}$	$\beta'_3 = \beta'_4 = \frac{55}{474}$
$\alpha_5 = \frac{-46}{869}$	$\alpha'_5 = \frac{112}{869}$	$\beta_5 = \frac{88}{237}$	$\beta'_5 = \frac{-70}{237}$
$\alpha_6 = \alpha_5/2$	$\alpha'_6 = \alpha'_5/2$	$\beta_6 = \beta_5/2$	$\beta'_6 = \beta'_5/2$

**B. Case 3**

In this case, we again have 13 conserved charges, as in Eq. (18), but with  $L_3 \rightarrow L_2$  ( $L'_3 \rightarrow L'_2$ ) in  $\mathcal{L}_2$  ( $\mathcal{L}'_2$ ), and  $L_2 \rightarrow L_3$ ,  $L'_2 \rightarrow L'_3$  in  $\mathcal{L}_0$ . The  $\alpha$  and  $\beta$  values are listed in Table II.

$$\mathcal{L}_4 = B_{u_{1R}} - B_{d_{1R}}, \quad \mathcal{L}'_4 = B'_{u_{1R}} - B'_{d_{1R}},$$

$$\mathcal{L}_5 = B_{d_{1R}} - B_{d_{2R}}, \quad \mathcal{L}'_5 = B'_{d_{1R}} - B'_{d_{2R}}. \quad (21)$$

In terms of these quantities,

**C. Cases 4 and 5**

Considering first case 4, at  $T \sim 10^{10}$  GeV, there are 12 conserved charges:

$$\mathcal{L}_0 = \frac{1}{3}B - L_3 + \frac{1}{3}B' - L'_2, \quad \mathcal{L}'_0 = \frac{1}{3}B' - L'_3 + \frac{1}{3}B - L_2,$$

$$B = \sum_{i=0}^5 (\alpha_i \mathcal{L}_i + \alpha'_i \mathcal{L}'_i),$$

$$L = \sum_{i=0}^5 (\beta_i \mathcal{L}_i + \beta'_i \mathcal{L}'_i). \quad (22)$$

$$\mathcal{L}_1 = \frac{1}{3}B - L_1, \quad \mathcal{L}'_1 = \frac{1}{3}B' - L'_1,$$

$$\mathcal{L}_2 = L_{e_{1R}}, \quad \mathcal{L}'_2 = L'_{e_{1R}},$$

$$\mathcal{L}_3 = L_{e_{2R}}, \quad \mathcal{L}'_3 = L'_{e_{2R}},$$

Under mirror symmetry,  $B \leftrightarrow B'$ ,  $L \leftrightarrow L'$ ,  $\mathcal{L}_i \leftrightarrow \mathcal{L}'_i$ . Hence,

$$B' = \sum_{i=0}^5 (\alpha_i \mathcal{L}'_i + \alpha'_i \mathcal{L}_i),$$

$$L' = \sum_{i=0}^5 (\beta_i \mathcal{L}'_i + \beta'_i \mathcal{L}_i). \quad (23)$$

TABLE III. The  $\alpha$  and  $\beta$  coefficients in the expansions of  $B$  and  $L$  for cases 4 and 5 as defined in Eqs. (22) and (23).

$D = 328321$			
$\alpha_0 = \frac{53484}{D}$	$\alpha'_0 = \frac{66132}{D}$	$\beta_0 = \frac{-123061}{D}$	$\beta'_0 = \frac{-85644}{D}$
$\alpha_1 = \frac{140874}{D}$	$\alpha'_1 = \frac{4908}{D}$	$\beta_1 = \frac{-177434}{D}$	$\beta'_1 = \frac{-5105}{D}$
$\alpha_2 = \frac{165018}{D}$	$\alpha'_2 = \frac{-34188}{D}$	$\beta_2 = \frac{72793}{D}$	$\beta'_2 = \frac{58037}{D}$
$\alpha_3 = \frac{90276}{D}$	$\alpha'_3 = \frac{14388}{D}$	$\beta_3 = \frac{164583}{D}$	$\beta'_3 = \frac{-59919}{D}$
$\alpha_4 = \frac{-48288}{D}$	$\alpha'_4 = \frac{78192}{D}$	$\beta_4 = \frac{156188}{D}$	$\beta'_4 = \frac{-126284}{D}$
$\alpha_5 = \frac{\alpha_4}{2}$	$\alpha'_5 = \frac{\alpha'_4}{2}$	$\beta_5 = \frac{\beta_4}{2}$	$\beta'_5 = \frac{\beta'_4}{2}$



TABLE IV. The  $\alpha$  and  $\beta$  coefficients in the expansions of  $B$  and  $L$  for case 6.

$D = 1343$		$D_2 \equiv 3D = 4029$	
	$\alpha_0 = \alpha'_0 = \frac{3519}{12D}$		$\beta_0 = \beta'_0 = \frac{-13617}{12D_2}$
$\alpha_1 = \frac{362}{D}$	$\alpha'_1 = \frac{46}{D}$	$\beta_1 = \frac{-2627}{D_2}$	$\beta'_1 = \frac{-178}{D_2}$
$\alpha_2 = \alpha_3 = \frac{4793}{12D}$	$\alpha'_2 = \alpha'_3 = \frac{1633}{12D}$	$\beta_2 = \beta_3 = \frac{25597}{12D_2}$	$\beta'_2 = \beta'_3 = \frac{-6319}{12D_2}$
$\alpha_4 = \frac{-637}{3D}$	$\alpha'_4 = \frac{943}{3D}$	$\beta_4 = \frac{4567}{3D_2}$	$\beta'_4 = \frac{-3649}{3D_2}$
$\alpha_5 = \frac{\alpha_4}{2}$	$\alpha'_5 = \frac{\alpha'_4}{2}$	$\beta_5 = \frac{\beta_4}{2}$	$\beta'_5 = \frac{\beta'_4}{2}$

The values of the  $\alpha$  and  $\beta$  parameters are given in Table III.

Results for case 5 follow from case 4 with the replacements  $L'_2 \rightarrow L'_1$  ( $L_2 \rightarrow L_1$ ) in  $\mathcal{L}_0$  ( $\mathcal{L}'_0$ ) and  $L_1 \rightarrow L_2$  ( $L'_1 \rightarrow L'_2$ ) in  $\mathcal{L}_1$  ( $\mathcal{L}'_1$ ). Because both muon- and electron-Higgs-boson Yukawa interactions are negligible above  $10^{10}$  GeV, the  $\alpha, \beta$  coefficients for case 5 are trivially the same as those of case 4 (given in Table III).

#### D. Case 6

This case is similar to case 4, except that  $L_1 \rightarrow L_3$  ( $L'_1 \rightarrow L'_3$ ) in  $\mathcal{L}_1$  ( $\mathcal{L}'_1$ ) and  $L_3 \rightarrow L_1$  ( $L'_3 \rightarrow L'_1$ ) in  $\mathcal{L}_0$  ( $\mathcal{L}'_0$ ). The  $\alpha, \beta$  coefficients are given in Table IV.

### IV. RESULTS AND CONCLUSIONS

The values of the conserved charges,  $\mathcal{L}_i$ ,  $\mathcal{L}'_i$  and  $\mathcal{L}_0$  depend on the *initial* asymmetry generation mechanism. As in Ref. [2], we consider, for definiteness, the simple case of non-zero  $B'$  and/or  $L'$ :  $B' = X'_0$ ,  $L' = Y'_0$ ,  $B = L = 0$  (with  $L'_{\ell_1} = L'_{\ell_2} = L'_{\ell_3} \equiv Y'_0/3$  and  $B'_{u_{1R}} = B'_{d_{1R}} = B'_{d_{2R}}$ ). The only nonzero conserved charges are then

$$\begin{aligned} \mathcal{L}'_1 = \mathcal{L}'_2 = \mathcal{L}'_3 = \frac{1}{3}(X'_0 - Y'_0) &\equiv Z, \quad \text{cases 1-3,} \\ \mathcal{L}'_1 = \mathcal{L}'_2 = \mathcal{L}'_3 &\equiv Z, \quad \text{cases 4-6.} \end{aligned} \quad (24)$$

After chemical processing, the baryon and mirror baryon asymmetries at  $T \approx 10^{10}$  GeV are then

$$\begin{aligned} B &= Z(\alpha_0 + \alpha'_1 + \alpha'_2), \quad B' = Z(\alpha_0 + \alpha_1 + \alpha_2), \quad \text{cases 1-3,} \\ B &= Z(\alpha_0 + \alpha'_0 + \alpha'_1), \quad B' = Z(\alpha_0 + \alpha'_0 + \alpha_1), \quad \text{cases 4-6.} \end{aligned} \quad (25)$$

The ordinary matter/dark matter ratio at that temperature is therefore

$$\frac{B}{B'} = \frac{\alpha_0 + \alpha'_1 + \alpha'_2}{\alpha_0 + \alpha_1 + \alpha_2} = \frac{3795}{15487} \approx 0.25, \quad \text{cases 1 and 2,}$$

$$\frac{B}{B'} = \frac{\alpha_0 + \alpha'_1 + \alpha'_2}{\alpha_0 + \alpha_1 + \alpha_2} = \frac{52}{289} \approx 0.18, \quad \text{case 3,}$$

$$\frac{B}{B'} = \frac{\alpha_0 + \alpha'_0 + \alpha'_1}{\alpha_0 + \alpha'_0 + \alpha_1} = \frac{20754}{43415} \approx 0.48, \quad \text{cases 4 and 5,}$$

$$\frac{B}{B'} = \frac{\alpha_0 + \alpha'_0 + \alpha'_1}{\alpha_0 + \alpha'_0 + \alpha_1} = \frac{1265}{1897} \approx 0.67, \quad \text{case 6.} \quad (26)$$

The value of this ratio changes to some extent at lower temperatures as different chemical processes become important. However, at temperatures near that of the electroweak phase transition,  $T = T_{EW} \sim 200$  GeV, the values of  $B$  and  $B'$  depend only on the values of  $B-L$  and  $B'-L'$ . These charges are separately conserved for  $T \ll 10^{10}$  GeV, because the interactions in Eqs. (5)–(8) chemically connecting the ordinary and mirror sectors are slower than the expansion rate. This yields the well-known relation between  $B$  and  $B-L$  [15],

$$B = \frac{28}{79}(B-L), \quad (27)$$

with an identical relation for  $B'$  in terms of  $B'-L'$ . Below the electroweak phase transition temperature, there are no processes fast enough to further affect  $B$  and  $B'$ . Thus the final, low temperature value of the ratio  $B/B' = \Omega_B/\Omega'_B$  is simply given by  $(B-L)/(B'-L')$  evaluated at  $T \approx 10^{10}$  GeV:

$$\begin{aligned} \frac{\Omega_B}{\Omega'_B} &= \frac{\alpha_0 + \alpha'_1 + \alpha'_2 - \beta_0 - \beta'_1 - \beta'_2}{\alpha_0 + \alpha_1 + \alpha_2 - \beta_0 - \beta_1 - \beta_2} = \frac{55}{256} \\ &\approx 0.22, \quad \text{cases 1 and 2,} \end{aligned}$$

$$\begin{aligned}
\frac{\Omega_B}{\Omega'_B} &= \frac{\alpha_0 + \alpha'_1 + \alpha'_2 - \beta_0 - \beta'_1 - \beta'_2}{\alpha_0 + \alpha_1 + \alpha_2 - \beta_0 - \beta_1 - \beta_2} = \frac{13}{53} \\
&\approx 0.25, \quad \text{case 3,} \\
\frac{\Omega_B}{\Omega'_B} &= \frac{\alpha_0 + \alpha'_0 + \alpha'_1 - \beta_0 - \beta'_0 - \beta'_1}{\alpha_0 + \alpha'_0 + \alpha_1 - \beta_0 - \beta'_0 - \beta_1} = \frac{214}{409} \\
&\approx 0.52, \quad \text{cases 4 and 5,} \\
\frac{\Omega_B}{\Omega'_B} &= \frac{\alpha_0 + \alpha'_0 + \alpha'_1 - \beta_0 - \beta'_0 - \beta'_1}{\alpha_0 + \alpha'_0 + \alpha_1 - \beta_0 - \beta'_0 - \beta_1} = \frac{55}{98} \\
&\approx 0.56, \quad \text{case 6.} \tag{28}
\end{aligned}$$

If there is some brief period of inflation between  $T \sim 10^{10}$  GeV and  $T_{EW}$ , as suggested by Step 4, then the results depend on when this second period of inflation occurs,  $T = T_3$ , as well as the subsequent ordinary and mirror sector reheating temperatures,  $T_{RH}$  and  $T'_{RH}$  respectively (with  $T'_{RH} < T_{RH}$  required for successful big bang nucleosynthesis and large scale structure formation).

At one extreme, if  $T_3 \sim T_{EW}$ , then the results of Eq. (28) hold irrespective of the reheating temperatures. The other extreme case,  $T_3 = 10^{10}$  GeV, allows three outcomes. If  $T_{RH} > T_{EW}$  and  $T'_{RH} > T_{EW}$ , then Eq. (28) again holds. In the opposite situation,  $T_{RH} < T_{EW}$  and  $T'_{RH} < T_{EW}$ , the final values are simply given by Eq. (26), because no further reprocessing can take place. The acceptable intermediate situation,  $T_{RH} > T_{EW}$  and  $T'_{RH} < T_{EW}$ , has further ordinary sector reprocessing but a frozen mirror sector. For this situation, the final ratios are given by

$$\begin{aligned}
\frac{\Omega_B}{\Omega'_B} &= \frac{28}{79} \frac{\alpha_0 + \alpha'_1 + \alpha'_2 - \beta_0 - \beta'_1 - \beta'_2}{\alpha_0 + \alpha_1 + \alpha_2} = \frac{3080}{15487} \\
&\approx 0.20, \quad \text{cases 1 and 2,} \\
\frac{\Omega_B}{\Omega'_B} &= \frac{28}{79} \frac{\alpha_0 + \alpha'_1 + \alpha'_2 - \beta_0 - \beta'_1 - \beta'_2}{\alpha_0 + \alpha_1 + \alpha_2} = \frac{182}{867} \\
&\approx 0.21, \quad \text{case 3,} \\
\frac{\Omega_B}{\Omega'_B} &= \frac{28}{79} \frac{\alpha_0 + \alpha'_0 + \alpha'_1 - \beta_0 - \beta'_0 - \beta'_1}{\alpha_0 + \alpha'_0 + \alpha_1} = \frac{1578892}{3429785} \\
&\approx 0.46, \quad \text{cases 4 and 5,} \\
\frac{\Omega_B}{\Omega'_B} &= \frac{28}{79} \frac{\alpha_0 + \alpha'_0 + \alpha'_1 - \beta_0 - \beta'_0 - \beta'_1}{\alpha_0 + \alpha'_0 + \alpha_1} = \frac{440}{813} \\
&\approx 0.54, \quad \text{case 6.} \tag{29}
\end{aligned}$$

These results all reproduce the qualitative observation that there is more dark matter than ordinary matter. Intuitively

this is because the Friedmann-Robertson-Walker universe is born full of mirror matter (under our assumptions), and only some of the net mirror baryon/lepton number is chemically reprocessed into an ordinary baryon asymmetry. But it is also interesting that there is a subset of effective dimension-5 operators which are *quantitatively* successful, namely cases 1–3 [with case 3 marginal unless the circumstances leading to Eq. (29) obtain]. Given that the effective operators also contribute to the light neutrino mass matrix through ordinary-mirror neutrino mixing, a tentative connection between the dark matter problem and neutrino oscillation physics can be made. The connection must be tentative, because some important assumptions lie behind our results, especially the microphysical desert between the electroweak scale and physics at  $10^{10}$ – $10^{12}$  GeV (as emphasized in Ref. [2]).

In conclusion, we have shown that the ratio of baryonic to nonbaryonic dark matter,  $\Omega_b/\Omega_{\text{dark}} = 0.20 \pm 0.02$ , inferred by WMAP [1], can be *quantitatively* explained if mirror matter is identified with the nonbaryonic dark matter. Our explanation involves a set of assumptions about the physics governing the early evolution of the Universe, which are not unique but are nevertheless plausible. Our approach also has important implications for neutrino physics, suggesting eV scale neutrino masses, which can be tested/constrained from upcoming neutrino experiments such as miniBooNE.

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## APPENDIX: SOLVING THE CASE 1 EQUATIONS

We show the algebraic technique for solving case 1. The other cases follow similarly.

The aim is to use Eqs. (11) and (12) to extract the total baryon number  $B$ , total lepton number  $L$ , and their mirror matter analogues  $B'$  and  $L'$  at  $T \approx 10^{10}$  GeV. These 15 equations reduce the 28 variables  $\mu_q, \mu_u, \mu_d, \mu_\ell, \mu_e, \mu_\phi$  and their primed counterparts to 13 independent variables. The number of independent variables corresponds to the number of conserved charges, Eq. (18). The problem at hand is to find  $B, L, B'$  and  $L'$  in terms of the conserved charges.

One systematic way of doing this is the following. First identify 13 independent variables. One possible choice is the following:  $\mu_q, \mu_{\ell_2}, \mu_{\ell_3}, \mu_{e_1}, \mu_{e_2}, \mu_{d_1}, \mu_{d_2}, \mu'_q, \mu'_{\ell_3}, \mu'_{e_1}, \mu'_{e_2}, \mu'_{d_1}$  and  $\mu'_{d_2}$ . Then use Eqs. (11) and (12) to write the 15 dependent  $\mu$  variables in terms of the chosen independent variables. Doing this we have

$$\begin{aligned}
\mu_{\ell_1} &= -9\mu_q - \mu_{\ell_2} - \mu_{\ell_3}, \\
\mu_\phi &= \frac{1}{6}(-21\mu_q + 3\mu_{d_1} + 3\mu_{d_2} + \mu_{e_1} \\
&\quad + \mu_{e_2} + \mu_{\ell_3}),
\end{aligned}$$

$$\begin{aligned}
\mu_{d_3} &= \frac{1}{6}(27\mu_q - 3\mu_{d_1} - 3\mu_{d_2} - \mu_{e_1} - \mu_{e_2} - \mu_{\ell_3}), \\
\mu_{e_3} &= \frac{1}{6}(21\mu_q - 3\mu_{d_1} - 3\mu_{d_2} - \mu_{e_1} - \mu_{e_2} + 5\mu_{\ell_3}), \\
\mu_{u_3} = \mu_{u_2} &= \frac{1}{6}(-15\mu_q + 3\mu_{d_1} + 3\mu_{d_2} + \mu_{e_1} + \mu_{e_2} + \mu_{\ell_3}), \\
\mu_{u_1} &= \frac{1}{6}(39\mu_q - 9\mu_{d_1} - 9\mu_{d_2} - \mu_{e_1} - \mu_{e_2} - \mu_{\ell_3}), \\
\mu'_{\ell_2} &= \mu_{\ell_2} + \mu_\phi - \mu'_\phi, \\
\mu'_{\ell_1} &= -9\mu'_q - \mu_{\ell_2} - \mu_\phi + \mu'_\phi - \mu'_{\ell_3}, \\
\mu'_\phi &= \frac{1}{6}(-21\mu'_q + 3\mu'_{d_1} + 3\mu'_{d_2} + \mu'_{e_1} + \mu'_{e_2} + \mu'_{\ell_3}), \\
\mu'_{d_3} &= \frac{1}{6}(27\mu'_q - 3\mu'_{d_1} - 3\mu'_{d_2} - \mu'_{e_1} - \mu'_{e_2} - \mu'_{\ell_3}), \\
\mu'_{e_3} &= \frac{1}{6}(21\mu'_q - 3\mu'_{d_1} - 3\mu'_{d_2} - \mu'_{e_1} - \mu'_{e_2} + 5\mu'_{\ell_3}), \\
\mu'_{u_3} = \mu'_{u_2} &= \frac{1}{6}(-15\mu'_q + 3\mu'_{d_1} + 3\mu'_{d_2} + \mu'_{e_1} + \mu'_{e_2} + \mu'_{\ell_3}), \\
\mu'_{u_1} &= \frac{1}{6}(39\mu'_q - 9\mu'_{d_1} - 9\mu'_{d_2} - \mu'_{e_1} - \mu'_{e_2} - \mu'_{\ell_3}),
\end{aligned} \tag{A1}$$

where it is understood that in the  $\mu'_{\ell_{1,2}}$  equations,  $\mu_\phi$  and  $\mu'_\phi$  are to be substituted with the respective right hand sides above. In terms of the chosen independent  $\mu_i$ , the baryon and lepton numbers are

$$\begin{aligned}
B &= 6\mu_q + \sum_{i=1}^3 (\mu_{u_i} + \mu_{d_i}) \\
&= 12\mu_q, \\
L &= \sum_{i=1}^3 (2\mu_{\ell_i} + \mu_{e_i}) \\
&= \frac{-29}{2}\mu_q - \frac{1}{2}\mu_{d_1} - \frac{1}{2}\mu_{d_2} + \frac{5}{6}\mu_{e_1} + \frac{5}{6}\mu_{e_2} \\
&\quad + \frac{5}{6}\mu_{\ell_3}.
\end{aligned} \tag{A2}$$

The next step in the calculation is to write the conserved charges,  $\mathcal{L}_i$  in Eq. (18), in terms of the 13 independent variables. For example,<sup>2</sup>

$$\begin{aligned}
\mathcal{L}_1 &= \frac{1}{3}B - L_1 \\
&= 2\mu_q + \frac{1}{3}\sum_{i=1}^3 (\mu_{u_i} + \mu_{d_i}) - 2\mu_{\ell_1} - \mu_{e_1} \\
&= 22\mu_q + 2\mu_{\ell_2} + 2\mu_{\ell_3} - \mu_{e_1},
\end{aligned} \tag{A3}$$

where Eq. (A1) has been used in the last step. The results for the other  $\mathcal{L}_i$  are given below:

$$\begin{aligned}
\mathcal{L}_0 &= 11\mu_q - 4\mu_{\ell_2} - \frac{4}{3}\mu_{e_2} - \frac{1}{3}\mu_{e_1} - \frac{1}{3}\mu_{\ell_3} - \mu_{d_1} - \mu_{d_2} - 3\mu'_q \\
&\quad + \mu'_{d_1} + \mu'_{d_2} + \frac{1}{3}\mu'_{e_1} - \frac{2}{3}\mu'_{e_2} + \frac{1}{3}\mu'_{\ell_3}, \\
\mathcal{L}_2 &= \frac{1}{6}(3\mu_q + 3\mu_{d_1} + 3\mu_{d_2} + \mu_{e_1} + \mu_{e_2} - 17\mu_{\ell_3}), \\
\mathcal{L}_3 &= \mu_{e_1}, \\
\mathcal{L}_4 &= \mu_{e_2}, \\
\mathcal{L}_5 &= \frac{1}{6}(39\mu_q - 15\mu_{d_1} - 9\mu_{d_2} - \mu_{e_1} - \mu_{e_2} - \mu_{\ell_3}), \\
\mathcal{L}_6 &= \mu_{d_1} - \mu_{d_2}, \\
\mathcal{L}'_1 &= 29\mu'_q - 7\mu_q + \mu_{d_1} + \mu_{d_2} + \frac{1}{3}\mu_{e_1} + \frac{1}{3}\mu_{e_2} + 2\mu_{\ell_2} + \frac{1}{3}\mu_{\ell_3} \\
&\quad - \mu'_{d_1} - \mu'_{d_2} - \frac{4}{3}\mu'_{e_1} - \frac{1}{3}\mu'_{e_2} + \frac{5}{3}\mu'_{\ell_3}, \\
\mathcal{L}'_2 &= \frac{1}{6}(3\mu'_q + 3\mu'_{d_1} + 3\mu'_{d_2} + \mu'_{e_1} + \mu'_{e_2} - 17\mu'_{\ell_3}), \\
\mathcal{L}'_3 &= \mu'_{e_1}, \\
\mathcal{L}'_4 &= \mu'_{e_2}, \\
\mathcal{L}'_5 &= \frac{1}{6}(39\mu'_q - 15\mu'_{d_1} - 9\mu'_{d_2} - \mu'_{e_1} - \mu'_{e_2} - \mu'_{\ell_3}),
\end{aligned}$$

<sup>2</sup>The conventional definition of, say, the “baryon number of the universe” is the ratio  $n_B/s$  where  $n_B$  is the net baryon number per unit volume, while  $s$  is entropy density. There is a proportionality factor relating this definition of baryon number to the simple one convenient for our application (just the appropriate linear combination of chemical potentials).



$$\mathcal{L}'_6 = \mu'_{d_1} - \mu'_{d_2}. \quad (\text{A4})$$

For the given set of values for the conserved quantities,  $\mathcal{L}_i$  and  $\mathcal{L}'_i$ , the baryon number can be found by solving the identities

$$\begin{aligned} B &= 12\mu_q = 12 \times \mu_q + 0 \times \mu_{\ell_2} + 0 \times \mu_{\ell_3} + \dots \\ &\equiv \alpha_0 \mathcal{L}_0 + \sum_{i=1}^6 (\alpha_i \mathcal{L}_i + \alpha'_i \mathcal{L}'_i), \end{aligned} \quad (\text{A5})$$

where it is understood that the  $\mathcal{L}'_i$ s are also functions of the 13 independent  $\mu$  variables [using Eq. (A4)]. By equating coefficients of each of the 13 independent variables a set of 13 simultaneous equations for the  $\alpha$  results:

$$\begin{aligned} 12 &= 11\alpha_0 + 22\alpha_1 + \frac{1}{2}\alpha_2 + \frac{13}{2}\alpha_5 - 7\alpha'_1, \\ 0 &= 2\alpha_1 - 4\alpha_0 + 2\alpha'_1, \\ 0 &= 2\alpha_1 - \frac{17}{6}\alpha_2 - \frac{1}{6}\alpha_5 - \frac{1}{3}\alpha_0 + \frac{1}{3}\alpha'_1, \\ 0 &= -\alpha_1 + \frac{1}{6}\alpha_2 + \alpha_3 - \frac{1}{6}\alpha_5 - \frac{1}{3}\alpha_0 + \frac{1}{3}\alpha'_1, \\ 0 &= \frac{1}{6}\alpha_2 + \alpha_4 - \frac{1}{6}\alpha_5 - \frac{4}{3}\alpha_0 + \frac{1}{3}\alpha'_1, \end{aligned}$$

$$0 = \frac{1}{2}\alpha_2 - \frac{5}{2}\alpha_5 + \alpha_6 - \alpha_0 + \alpha'_1,$$

$$0 = \frac{1}{2}\alpha_2 - \frac{3}{2}\alpha_5 - \alpha_6 - \alpha_0 + \alpha'_1,$$

$$0 = -3\alpha_0 + 29\alpha'_1 + \frac{1}{2}\alpha'_2 + \frac{13}{2}\alpha'_5,$$

$$0 = \frac{1}{3}\alpha_0 + \frac{5}{3}\alpha'_1 - \frac{17}{6}\alpha'_2 - \frac{1}{6}\alpha'_5,$$

$$0 = \frac{1}{3}\alpha_0 - \frac{4}{3}\alpha'_1 + \frac{1}{6}\alpha'_2 + \alpha'_3 - \frac{1}{6}\alpha'_5,$$

$$0 = \frac{-2}{3}\alpha_0 - \frac{1}{3}\alpha'_1 + \frac{1}{6}\alpha'_2 + \alpha'_4 - \frac{1}{6}\alpha'_5,$$

$$0 = \alpha_0 - \alpha'_1 + \frac{1}{2}\alpha'_2 - \frac{5}{2}\alpha'_5 + \alpha'_6,$$

$$0 = \alpha_0 - \alpha'_1 + \frac{1}{2}\alpha'_2 - \frac{3}{2}\alpha'_5 - \alpha'_6. \quad (\text{A6})$$

These 13 equations can easily be solved for the 13  $\alpha$ 's; the results are as displayed in Table I.

A similar procedure with  $L$  instead of  $B$  in Eq. (A5) yields results for the  $\beta$ 's.

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- [1] WMAP Collaboration, D.N. Spergel *et al.*, *Astrophys. J., Suppl. Ser.* **148**, 175 (2003), and references therein for earlier work.
- [2] R. Foot and R.R. Volkas, *Phys. Rev. D* **68**, 021304 (2003).
- [3] L. Bento and Z. Berezhiani, hep-ph/0111116; see also *Phys. Rev. Lett.* **87**, 231304 (2001).
- [4] T.D. Lee and C.N. Yang, *Phys. Rev.* **104**, 256 (1956); I. Kobzarev, L. Okun, and I. Pomeranchuk, *Sov. J. Nucl. Phys.* **3**, 837 (1966); M. Pavsic, *Int. J. Theor. Phys.* **9**, 229 (1974); S.I. Blinnikov and M.Yu. Khlopov, *Sov. J. Nucl. Phys.* **36**, 472 (1982); *Sov. Astron.* **27**, 371 (1983).
- [5] R. Foot, H. Lew, and R.R. Volkas, *Phys. Lett. B* **272**, 67 (1991).
- [6] B. Holdom, *Phys. Lett.* **166B**, 196 (1985); S.L. Glashow, *ibid.* **167B**, 35 (1986); E.D. Carlson and S.L. Glashow, *Phys. Lett. B* **193**, 168 (1987); R. Foot and X.-G. He, *ibid.* **267**, 509 (1991); S.N. Gninenko, *ibid.* **326**, 317 (1994); R. Foot and S. Gninenko, *ibid.* **480**, 171 (2000); A.Yu. Ignatiev and R.R. Volkas, *ibid.* **487**, 294 (2000); R. Foot and R.R. Volkas, *ibid.* **517**, 13 (2001); R. Foot, A.Yu. Ignatiev, and R.R. Volkas, *ibid.* **503**, 355 (2001); R. Foot, *Acta Phys. Pol. B* **32**, 3133 (2001); R. Foot and T.L. Yoon, *ibid.* **33**, 1979 (2002); R. Foot and S. Mitra, *Phys. Rev. D* **66**, 061301 (2002); *Astropart. Phys.* **19**, 739 (2003); *Phys. Lett. B* **558**, 9 (2003); *Phys. Lett. A* **315**, 178 (2003); *Phys. Rev. D* **68**, 071901 (2003); Z. Silagadze, astro-ph/0311337; R. Foot, *Phys. Rev. D* **69**, 036001 (2004); astro-ph/0309330.
- [7] Z. Berezhiani, D. Comelli, and F.L. Villante, *Phys. Lett. B* **503**, 362 (2001); A.Yu. Ignatiev and R.R. Volkas, *Phys. Rev. D* **68**, 023518 (2003); Z. Berezhiani, P. Ciarcelluti, D. Comelli, and F.L. Villante, astro-ph/0312605; P. Ciarcelluti, astro-ph/0312607.
- [8] E.W. Kolb, D. Seckel, and M.S. Turner, *Nature (London)* **314**, 415 (1985); H.M. Hodges, *Phys. Rev. D* **47**, 456 (1993); Z.G. Berezhiani, A.D. Dolgov, and R.N. Mohapatra, *Phys. Lett. B* **375**, 26 (1996); V. Berezhinsky and A. Vilenkin, *Phys. Rev. D* **62**, 083512 (2000).
- [9] M. Fukugita and T. Yanagida, *Phys. Lett. B* **174**, 45 (1986).
- [10] C. Athanassopoulos *et al.*, *Phys. Rev. C* **54**, 2685 (1996); *Phys. Rev. Lett.* **81**, 1774 (1998); A. Aguilar *et al.*, *Phys. Rev. D* **64**, 112007 (2001); see also G. Drexlin, *Nucl. Phys. B (Proc. Suppl.)* **118**, 146 (2003).
- [11] R. Foot, H. Lew, and R.R. Volkas, *Mod. Phys. Lett. A* **7**, 2567 (1992); R. Foot, *ibid.* **9**, 169 (1994); R. Foot and R.R. Volkas, *Phys. Rev. D* **52**, 6595 (1995).
- [12] R.N. Mohapatra and X. Zhang, *Phys. Rev. D* **45**, 2699 (1992).
- [13] V.A. Kuzmin, V.A. Rubakov, and M.E. Shaposhnikov, *Phys. Lett.* **155B**, 36 (1985).

- [14] See W. Buchmüller, in *Cascais 2000: Recent Developments in Particle Physics and Cosmology*, pp. 281–314, hep-ph/0101102, for a review.
- [15] S.Yu. Khlebnikov and M.E. Shaposhnikov, Nucl. Phys. **B308**, 885 (1988). There are small higher-order corrections to this result; see S.Yu. Khlebnikov and M.E. Shaposhnikov, Phys. Lett. B **387**, 817 (1996); M. Laine and M.E. Shaposhnikov, Phys. Rev. D **61**, 117302 (2000).